Correspondence

Comments on "A Computational Evolutionary Approach to Evolving Game Strategies and Cooperation"

Hsu-Chih Wu and Chuen-Tsai Sun

***Abstract*—Azuaje offers an approach to the co-evolution of competing virtual creatures and a model for the evolution of game strategies and their emerging behaviors [1]. This model can be greatly simplified and optimal solutions can be obtained more quickly and easily by using an analytical approach. We emphasize the importance of performing a model analysis before choosing an evolutionary or analytical approach to a problem. Furthermore, Azuaje's model is derived from the Prisoner's Dilemma, a classical model in game theory; some results have already been discussed in the literature. We discuss his model from the perspective of game theorists.**

***Index Terms*—Artificial life, evolutionary computation, genetic algorithm, prisoner's dilemma.**

I. Introduction

The emergence of cooperation is currently the focus of many research papers in economics, biology, computer science, and sociology [2]. In [1], Azuaje proposes an approach to the co-evolution of competing virtual creatures to model the emergence of cooperation in game strategy. According to this model, both: 1) cooperative behavior can emerge from an evolutionary and unsupervised learning process and 2) evolving organisms are able to achieve individual success by developing strategies that are more effective than Tit-for-Tat.

In this paper, we will argue that this model can be greatly simplified and offer an alternative analytical approach that we feel is more efficient. We believe that our simplified model underscores the complementary properties of evolutionary and analytical approaches. The robustness of the evolutionary approach makes it very popular among researchers in various disciplines; they find it easy to use for formulating problems and obtaining solutions. However, two important considerations are frequently overlooked by the users of the evolutionary approach: the importance of pre-run model analysis and the need to make a conscious decision between evolutionary and analytical approaches to individual problems based on their specific characteristics.

Furthermore, in Azuaje's mode, interactions between two types of organisms is a clear example of an Iterated Prisoner's Dilemma (IPD) game. Even though Azuaje claims that his model does not implement a traditional Prisoner's Dilemma system, some results of [1]'s model is similar to the studies published by Axelrod [2], and have already been discussed. We will extend this discussion of the model and its results from the perspective of game theorists.

II. An Analytical Approach to the Model

In [1], the artificial life model considers two kinds of organisms, X and Y. Their individual decisions about whether or not to approach a food source are presented in the form of an IPD game. Y is represented by a genetic code that determines its sequence of moves against X. Y cannot recognize individuals or store (remember) previous events. In contrast, X is a more sophisticated organism that can perform basic cognitive and memory functions; it acts in accordance with a standard Tit-for-Tat strategy, mimicking each move that its opponent made in the preceding round.

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TABLE I
Moves of $\mathbf{y}$ and X From Round 1 to Round n

	1	**2**	**…**	***n***
y	s_1	s_2	…	s_n
X	0	s_1	…	s_{n-1}

TABLE II
Potentail Scores of $\mathbf{y}$ for Each Combination of $(a_X, a_{\mathbf{y}})$

(a_X, a_y)	(0, 0)	(0, 1)	(1, 0)	(1, 1)
$\mathbf{y}$'s score $P(a_X, a_y)$	R	T	S	P

Y-type organisms evolve according to two basic genetic algorithm (GA) operators: crossover and mutation. Even though there are two kinds of organisms, only Y is subject to the forces of artificial selection. X, whose primary function is to evaluate Y's score, cannot evolve. We therefore believe it is unnecessary to select "100 fittest individuals from each type of organism to be included in the next generation" [1]; that selection process can be limited to Y-type organisms.

[1]'s model can be formulated to the following problem: find an optimal binary string $\mathbf{y}_{\mathrm{opt}}$ that maximizes $\mathrm{F}(\mathbf{y})$, where the binary string $\mathbf{y}$ represents a Y-type organism; 0's encoded in $\mathbf{y}$ represent "do not approach food" and 1's represent "approach food"; and a fitness function $\mathrm{F}(\mathbf{y})$ denotes a $\mathbf{y}$ score when it plays with X (the Tit-for-Tat strategy) n times, with n equal to the length of $\mathbf{y}$.

Let $\mathbf{y} = s_1 s_2 s_3 \ldots s_n$. $\mathbf{y}$'s moves from round 1 to round n are expressed as

$$s_1, s_2, s_3, \ldots, s_{n-1}, s_n.$$

Since X simply repeats its opponent's preceding moves, its moves are expressed as

$$0, s_1, s_2 \ldots, s_{n-2}, s_{n-1}.$$

Moves of $\mathbf{y}$ and X from round 1 to round n are shown in Table I.

The interaction between $\mathbf{y}$ and X during a single round is represented as

$$(a_X, a_{\mathbf{y}})$$

where a_X stands for X's move, and $a_{\mathbf{y}}$ stands for $\mathbf{y}$'s move. There are four possible combinations for $(a_X, a_{\mathbf{y}})$. For each combination, $\mathbf{y}$ receives a score (in IPD terminology, a payoff) $\mathrm{P}(a_X, a_{\mathbf{y}})$. Possible scores are listed in Table II.

In Table II, $[R, S, T, P]$ denotes four IPD payoff values. The payoff matrix and value constraints for IPD game is listed in Table III. In [1]'s model, $[R, S, T, P] = [3, 0, 5, 1]$.

TABLE III
PAYOFF MATRIX AND VALUE CONSTRAINTS FOR IPD MODEL

Myself\Oppoent	Cooperation	Defection
Cooperation	R\R	S\T
Defection	T\S	P\P

Note: $\mathrm{T} > \mathrm{R} > \mathrm{P} > \mathrm{S}$, and $2\mathrm{R} > \mathrm{S} + \mathrm{T}$.

An interaction history of $\mathbf{y}$ and X moves from round 1 to round n is denoted as a sequence of $(\mathrm{a_X}, \mathrm{a_y})$ pairs

$$(0, \mathrm{s}_1), (\mathrm{s}_1, \mathrm{s}_2), (\mathrm{s}_2, \mathrm{s}_3), (\mathrm{s}_3, \mathrm{s}_4), \ldots, (\mathrm{s}_{n-1}, \mathrm{s}_n).$$

The problem can be reformulated as follows. Find a binary string $\mathbf{y} = \mathrm{s}_1\mathrm{s}_2\mathrm{s}_3 \ldots \mathrm{s}_n$, to maximize

$$\sum_{m=1}^{n} P(\mathrm{s}_{m-1}, \mathrm{s}_m)$$
$$\text{where } \mathrm{s}_0 = 0$$
$$P(0,0) = 3$$
$$P(0,1) = 5$$
$$P(1,0) = 0$$
$$P(0,0) = 1.$$

To solve this problem, define B_m as a pattern with m consecutive 1's sandwiched between two 0's, that is

$$\mathrm{B}_1 = 010$$
$$\mathrm{B}_2 = 0110$$
$$\ldots$$
$$\mathrm{B}_{n-1} = 011\ldots10, \text{ with } (n-1)1'\text{s}.$$

The longest pattern is represented as B_{n-1}. Since n is the length of $\mathbf{y}$, after adding s_0 at the beginning of $\mathbf{y}$, the maximum number of consecutive 1's must be $(n-1)$ in order to satisfy the constraint that it starts and ends with 0.

Let the number of $\mathrm{B}_1, \mathrm{B}_2, \ldots, \mathrm{B}_{n-1}$ patterns in $\mathbf{y}$ be $\mathrm{b}_1, \mathrm{b}_2, \ldots, \mathrm{b}_{n-1}$. Most strings with a random combination of 0's and 1's can be represented as the $\mathrm{b}_1, \mathrm{b}_2, \ldots,$ and b_{n-1} of $\mathrm{B}_1, \mathrm{B}_2, \ldots$ and B_{n-1} patterns connected by arbitrary numbers of 0's (Fig. 1). Strings that start or end with 1 cannot be represented in this form. These cases will be addressed later in this section.

The reason for choosing B_m patterns to represent binary strings is to compute $\mathbf{y}$'s score. In cases of consecutive 0's, the $\mathbf{y}$ score will be the sum of consecutive R's. At the appearance of the first 1, the score of that round changes from R to T, since $(\mathrm{a_X}, \mathrm{a_y})$ is (0, 1). The score of subsequent round will be S if that 1 is followed by a 0 and P if it is followed by another 1. B_m pattern scores for $m = 1$ to $(n-1)$ are listed in Table IV. They can be formulated as

$$\mathrm{PP}(\mathrm{B}_m) = (T + (m-1)P + S).$$

$\mathbf{y}$'s score can be computed by adding the scores of consecutive 0's plus the summation of B_m patterns' scores from $m = 1$ to $(n-1)$. However, this kind of representation does not work when the string ends with 1. Therefore, two cases must be considered.

Case 1) Strings that end with 0 whose regular expression is

$$0\{1,0\}^*0.$$

Case 2) Strings that end with x number of consecutive 1's, whose regular expression is

$$0\{1,0\}^*1^+.$$

... 0 [0 1 ... 1 0] 0 ... 0 [0 1 ... 1 0] 0 ... 0 [0 1 ... 1 0] 0 ...
B_i B_j B_k

Fig. 1. String combinations can be represented as B_m patterns connected by consecutive 0's.

TABLE IV
B_m SCORES FOR $m = 1$ TO $(n-1)$

$\mathbf{B}_m$	$\mathbf{B}_1$	$\mathbf{B}_2$	$\mathbf{B}_3$	...	$\mathbf{B}_{n\text{-}1}$
Encoding	010	0110	01110	...	011...10
Interaction History	(0,1), (1,0)	(0,1), (1,1), (1,0)	(0,1), (1,1), (1,1), (1,0)		(0,1), (1,1), ..., (1,1), (1,0)
Score	T+S	T+P+S	T+2P+S	...	T+(n-2)P+S

Note that the cases that strings start with 1 are skipped since $\mathrm{s}_0 = 0$ in our problem formulation.

The $\mathbf{y}$ score: $\sum_{m=1}^{n} P(\mathrm{s}_{m-1}, \mathrm{s}_m)$ can be accumulated as follows.

Case 1) The string is divided into two parts: the B_1 to B_{n-1} patterns and the consecutive 0's that connect them. Thus

$$\begin{aligned}
&\sum_{m=1}^{n} P(\mathrm{s}_{m-1}, \mathrm{s}_m) \\
&= \sum (\mathrm{B}_1 \text{to } \mathrm{B}_{n-1} \text{score}) + \sum (\text{Consecutive } 0'\text{s score}) \\
&= \sum_{m=1}^{n-1} (PP(B_m) \times b_m) + \left(\left(n - \sum_{m=1}^{n-1} ((2 + (m-1)) \times b_m \right) \times R \right) \\
&= (T + S) \times b_1 + (T + P + S) \times b_2 + (T + 2P + S) \times b_3 \\
&\quad + \cdots + (T + ((n-1) - 1)P + S) \times b_{n-1} \\
&\quad + ((n - 2(b_1 + b_2 + \cdots + b_{n-1}) \\
&\quad - (b_2 + 2b_3 + \cdots + (n-1-1)b_{n-1})) \times R \\
&= n \times R + (T + S - 2R) \times \sum_{m=1}^{n-1} b_m + (P - R) \sum_{m=2}^{n-1} ((m-1) \times b_m).
\end{aligned}$$

Case 2) The string includes an additional section consisting of the last x consecutive 1's, where $x \geq 1$. Thus

$$\begin{aligned}
&\sum_{m=1}^{n} P(\mathrm{s}_{m-1}, \mathrm{s}_m) \\
&= \sum (\mathrm{B}_1 \text{ to } \mathrm{B}_{n-1} \text{ score}) + \sum (\text{Consecutive } 0'\text{s score}) \\
&\quad + \sum (\text{Last consecutive } 1'\text{s score}) \\
&= \sum_{m=1}^{n-1} (PP(B_m) \times b_m) + \left(\left(n - x - \sum_{m=1}^{n-1} ((2 + (m-1)) \times b_m \right) \times R \right) \\
&\quad + (T + (x-1) \times S) \\
&= (T + S) \times b_1 + (T + P + S) \times b_2 + (T + 2P + S) \times b_3 \\
&\quad + \cdots + (T + ((n-1) - 1)P + S) \times b_{n-1} \\
&\quad + ((n - x - 2(b_1 + b_2 + \cdots + b_{n-1}) \\
&\quad - (b_2 + 2b_3 + \cdots + (n-1-1)b_{n-1})) \times R \\
&\quad + T + (x-1) \times S \\
&= n \times R + (T + S - 2R) \times \sum_{m=1}^{n-1} b_m + (P - R) \sum_{m=2}^{n-1} ((m-1) \times b_m) \\
&\quad + (T + (x-1) \times S - x \times R).
\end{aligned}$$

Based on constraint of IPD: $2R > T + S$ and $R > P$, maximizing

$$(T + S - 2R) \times \sum_{m=1}^{n-1} b_m + (P - R) \sum_{m=2}^{n-1} ((m-1) \times b_m)$$

requires that $\mathrm{b}_1, \mathrm{b}_2, \ldots$, and b_m be 0. The result is as follows.

Case 1) $\sum_{m=1}^{n} P(\mathrm{s}_{m-1}, \mathrm{s}_m) = n \times R$.

Case 2) $\sum_{m=1}^{n} P(\mathrm{s}_{m-1}, \mathrm{s}_m) = n \times R + (T + (x-1) \times S - x \times R)$.

In Case 2, since $[R, S, T, P] = [3, 0, 5, 1]$

$$(T + (x-1) \times S - x \times R) = 5 - 3\,x$$

which is greater than 0 if $x < 5/3$. Since $x \geq 1$, the optimal $\mathbf{y}_{\mathrm{opt}}$ is obtained when $x = 1$

$$\mathbf{y}_{\mathrm{opt}} = 0000\ldots\ldots001$$

with a score of

$$\sum_{m=1}^{n} P(\mathrm{s}_{m-1}, \mathrm{s}_m) = n \times R + (T - R).$$

If $n = 30$, then

$$\mathbf{y}_{\mathrm{opt}} = 000\,000\,000\,000\,000\,000\,000\,000\,000\,001$$

with a score of

$$\sum_{m=1}^{n} P(\mathrm{s}_{m-1}, \mathrm{s}_m) = (30 \times 3 + (5 - 3)) = 92.$$

This result is exactly the same as that produced by [1]'s evolutionary approach. The mathematical procedure implemented to obtain optimal y score: $\sum_{m=1}^{n} P(\mathrm{s}_{m-1}, \mathrm{s}_m)$ is shown in Fig. 2.

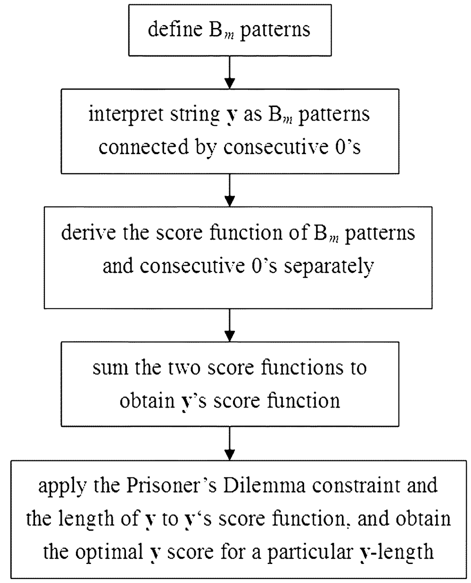

Fig. 2. Mathematical procedure for obtaining optimal $\mathbf{y}$ score.

III. Discussion

Our approach emphasizes an important question: What should be done before using evolutionary approach to solve a problem? We believe that too many researchers overlook the importance of model analysis. The model in the current example is complex on the surface because it contains two kinds of organisms. However, since X cannot evolve, it only serves as an evaluation tool for Y—that is, the problem actually addresses only one kind of organism. When using an evolutionary approach, only Y organisms need to be selected for the next generation, cutting the number of artificial selection operations in half and drastically reducing computation time. Our main point here is that pre-run analytical work can increase one's understanding of problem scope, reduce model complexity, and help in the search for appropriate parameters—in short, increase the efficiency of a search for appropriate solutions.

Our result was the same as that of [1], but our analytical approach was faster and simpler. In this model, our approach is appropriate in cases with different Y lengths, but the constraints of the evolutionary approach dictate that a GA must be run for each change in the length of Y. We are not claiming that an analytical approach is always superior to an evolutionary approach, since they clearly have complementary advantages and disadvantages. An analytical approach is much less effective than an evolutionary approach when problem spaces exceed a certain size or complexity threshold. This was not the case in the example of [1].

Next, we discuss the model results from a game theorists' perspective. In [1], Azuaje states that Y was able to achieve individual success "by learning to approach the source at the end of a contest." This strategy was more successful than Tit-for-Tat. He also makes the claim that "information about the length of the games was not provided to the creature." We believe game length was implied in Y's encoding, since n is the length of Y and Y always plays with X n times. Thus, the important "shadow of the future" assumption of stable cooperation no longer holds [2]; in Axelrod's words, "if you are unlikely to meet the other person again, or if you care little about future payoffs, then you might as well defect now and not worry about the consequences for the future" [2]. The behavior of an evolved solution for [1]'s model has already been discussed and verified in the literature [2].

Azuaje also wrote that "the emergence of cooperation did not require special assumptions about the individuals and the game environment." We suggest that the emergence of cooperation is actually determined by the X organism, which uses the Tit-for-Tat strategy in his model. The strategy encourages the evolution of Y toward a strategy that is equal to or better than Tit-for-Tat, which in turn encourages mutual cooperation. Assuming that X follows an "always defect" strategy (ALLD), then Y will also defect, and cooperative behavior will not emerge.

Furthermore, it is generally accepted that no evolutionarily stable strategy (ESS) exists for traditional IPD games, meaning that no prevalent strategy exists for extended IPD interactions [3]. Game theorists are less concerned with finding a dominant IPD strategy than with investigating relationships among strategies [5] and identifying conditions under which strategies become evolutionarily stable [4]. The model in [1] would be very interesting if X used more than one strategy or if X were also capable of evolving. Either case would result in complex evolutionary dynamics, underscoring the weaknesses of the analytical approach and emphasizing the strengths of the evolutionary approach.

IV. Conclusion

Our analytical approach to the problem described in [1] is a faster and easier alternative to the evolutionary approach. In this paper, we also emphasized two important considerations that are frequently overlooked by users of the evolutionary approach: the importance of pre-run model analysis and the need to make a conscious decision between an evolutionary or analytical approach to solving a problem. We also addressed some game theory considerations regarding the Prisoner's Dilemma and other two-person matrix games.

Our approach can be expanded in order to solve more sophisticated problems. For example, in cases where X-type organisms use other kinds of strategies, y's score function may change; at a certain level of sophistication for X and y encoding, interactions between them may become too complex to be represented as a string. In such cases, a finite state machine representation may be useful for representing interactions between the two strategies [5]. Furthermore, our approach can be applied to other form of two-person matrix games (e.g., chicken games), and Prisoner's Dilemma derivatives (e.g., N-person and N-choice Prisoner's Dilemma). It is also important to investigate the extent to which the analytical approach is useful for nondeterministic strategies (strategies with slight chances of deviations in moves). For those sophisticated models, we believe a combination of analytical and evolutionary approaches may be more efficient than relying on either one alone. Further investigation is required to clarify the complementary properties of the two approaches.

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